ASSIGNMENT SET - I

Department of Mathematics

Mugberia Gangadhar Mahavidyalaya

B.Sc Hon.(CBCS)

Mathematics: Semester-V

Paper Code: DSE2T

[Probability & Statistics]

Answer all the questions

1) (i) Define joint distribution $F(x, y)$. Prove that a necessary and sufficient condition for two random variables Xand Y to be independent is that $F(x, y)$ can be expressed a $F(x, y) = f(x).\emptyset(y)$

(ii) If X_1^2 , X_2^2 are independent variates having chi-square distribution with m , n degree of freedom respectively , then prove that $\frac{nX_1^2}{m\,X_2^2}$ is a veriate .

(iii) If the probability density function of a random variable X is given by $f(x) = c\mathrm{e}^{-(x^2+2x+3)}, \; -\infty < x < \infty$ find the value of c , the expectation and variance of the distribution .

2) (i) A problem in mathematics is given to (n – 1) students whose chances of solving it are respectively $\frac{1}{2}, \frac{1}{3}, \ldots \frac{1}{n}$. What is the probability that the problem will be solved.

(ii) The ray of light is sent in a random direction towards x - axis from the point Q (0 , 1) on y -axis and the ray meets x - axis at a point P. Find the distribution of the abscissae of P.

(iii) Prove that for normal (u , σ) distribution u_{2r+1} =0 and u_{2r} = 1. 3. 5 … $(2r-1)\sigma^{2r}$, r being positive integer.

3) (i) By the method of characteristic function show that a X^2 – variate with n degrees of freedom

asymptotically normal (n ,$\sqrt{2n}$) variate .

(ii) If the mutually independent random variables $X_1, X_2, \ldots, X_n$ all have the same distribution and their sum $X_1 + X_2 + \ldots + X_n$is normally distributed then show that each of them is normally distributed then show that each of them is normally distributed .

4) (i) Show that variance of t- distribution with n degrees of freedom exists for n>2 and hence obtain its value .

(ii) A continuous distribution has probability density function f(x) = ae^{-ax} , 0< x<∞ , a> 0 Calculate moment generating function and hence obtain a_k .

5) Let X and Y are independent , $\gamma(l)$ and $\gamma(m)$ variates respectively . Prove that $X + Y$ and $\frac{X}{X+Y}$ are independent ,$\gamma(l+m)$ and $\beta(l,m)$ variates respectively .

6) State and prove chebyshev's inequality .

7) (i) Example the term random sample and write the difference between sampling distribution and distribution of sample .

(ii) Find the sampling distribution of the static

$\tau = \frac{\sqrt{n}\,(\bar{x}-m)}{s}$, where $\bar{x}$ is the sample mean and (n-1)s² $=\sum_{i=1}^{n}(x_i - \overline{x})$. ($x_1, x_2, \ldots, x_n$)represent a sample of size n from a normal population of mean m and variance σ^2)

(iii) Show that regression coefficients independent of change of origin but depends on change of scales of variable.

8) (i) Apply the method of testing the $hypothens H_0$: p=p_0 for binomial (n, p) population , when n is known and large.

(ii) Define likelihood function. Find the maximum likelihood estimate of the parameter λ of a continuous population having the density function

$F(x) = \lambda x^{\lambda-1}$, 0 <$x$<$1$ where $\lambda > 0$.

(iii) In a group of 10 students , a dull student secure 25 marks below the average marks of the other student .Prove that the student deviation of marks of all is at least 7.5 .If this standard deviation when the dull student is left out.

9) (i) The heights in inches of 7 student of a college , chosen at random were as follows : 65.3,68.4,68.2, 64.2, 62.3,64.5,61.5

Compute 95% confidence intervals for the mean and standard deviation of the population of the height of the students of the college, assuming it to be normal.

Given p (t>2.447) =0.025,P(x²>1.2180) =0.975 P(x²>14.626) =0.025 For 6 degree of freedom.

(ii) Distinguish between a population and sample.

10) If r be the sample correlation coefficient of a $bivariate$ sample (x_1, y_1), (x_2, y_2),…,(x_n, y_n), then prove that $-2 \leq r \leq 1$, also prove that the sample correlation coefficient is independent of the unit of measurement and of the choice of the origin .Give signification for r=0.

11)State $Neymann$ Pearson theorem on the best critical region of a test of hypothesis.

12)Show that sample mean is consistent estimates of population mean.

13) Show that F(x) = $\begin{cases} 0, -\infty < x < \infty \\ 1 - e^{x}, 0 \leq x < \infty \end{cases}$ is a possible distribution function .

14) If X is uniformly distributed in the interval (-1 ,1) , find the distribution of $|x|$.

15) Prove that a necessary and sufficient condition for the independence of the random variables X and Y is that their joint distribution function F (x , y) can be written as the product of a function of x alone and a function of y alone .

16) Show that P (a $\leq x \leq b$) = F (b) – F (a) , where F(x) denotes the distribution function of a random variable .

17) If X is a normal (m , σ) variable , prove that P (a $< x < b$) = $\emptyset\left(\frac{b-m}{\sigma}\right) - \emptyset\left(\frac{a-m}{\sigma}\right)$.

18) If X is standard normal variate , then show that $\frac{1}{2}x^2$ is $\gamma\left(\frac{1}{2}\right)$ variate .

19) Two numbers are independently chosen at random between 0 and 1 . Show that the probability of their product is less than a constant k ($0 < k < 1$) is k ($1 - \log k$) .

20) Let , X and Y are independent $\gamma(l)$ and $\gamma(m)$ variates respectively .

21) Show that U= X+Y and V= $\frac{x}{X+Y}$ are independent $\gamma(l+m)$ and $B_1(l,m)$ variates .

If X is poisson distributed with parameter μ, then prove that $P(X \leq n) = \frac{1}{n!}\int_{\mu}^{\infty} e^{-x}x^{n}\,dx$,where n is any positive integer .

22) Find the density function of $F(x) = \begin{cases} 0, -\infty < x < \infty \\ 1 - e^{-x}, 0 \leq x < \infty \end{cases}$.

23) State Bernoullies theorem.

24) State and prove Bay's theorem .

25) Define Axiomatic definition of probability.

26) Give the expression of Frequency interpretation.

27) From an urn containing n balls any numbers of balls are drawn . Show that the Probability of drawing an event number of balls is $\frac{2^{n-1}-1}{2^{n}-1}$.

28) If $\{A_n\}$ be a monotonic sequence of events , then show that

$$P(\lim A_n) = \lim P(A_n)$$

29) Proved that the conditional probability satisfy the Axioms of probability .

30) If x is a random variable uniformly distributed over the interval (0,2) , find the distribution function of the large root of the equation $t^2+2t-x=0$.

31) If x_1 and x_2 are independent random variables each having the density function: f(x)= $2xe^{-x^2}$, $0< x < \infty$. Find the density function for the $random\ variable\ \sqrt{x_1^2+x_2^2}$.

32) Let, x be a continuous random variable having distribution function F(x).

Show that y=F(x) has uniformly $distribution\ \ over$ (0,1).

33) Show that F(x)= $\begin{cases} 0, -\infty < x < 0 \\ 1-e^{-x}, 0 \le x < \infty \end{cases}$ is a possible distribution function, and find the density function .

34) Let X and Yare independent $\gamma(1) and\ \gamma(m)$variates respectively. Show that U= X+Y and V= $\frac{X}{X+Y}$ are independent $\gamma(l+m)$ and B(l,m) variates.

35) If X is position distributed with parameterμ, then prove that

P(X$\le n)$ =$\frac{1}{n!}\int_{\mu}^{\infty} e^{-x}x^n$ dx , where n is any positive integer.

36) Prove that under certain condition (to be stated by you), the number of telephone calls on a trunk line in a given interval of time ha a Poisson distribution.

37) Show that the first absolute moment about the mean for the normal (m,σ) distribution is $\sqrt{\frac{2}{\pi}}\sigma$.

38) A continuous distribution has probability density f(x) =ae^{-ax} (0< x< ∞; a>0) . Obtain the moment generating function and hence obtain 'a' .

39) The probability density function of a random variable X is given by f(x) = c$e^{-(x^2+2x+3)}$, -∞< x< ∞ . Find the expectation of the distribution .

40) Find the median and mode of a distribution having the density function $\lambda e^{-\lambda x}$ (x>0).

41) A moment generating function of a uniform distribution over the interval $(-a, a)$ is $\frac{sinhat}{at}$. Hence calculate the central moment .

42) Express the mean of the $isson\ distribution$. For the $poisson$ distribution with parameter λ , prove that μ_{k+1} = λ (k μ_{k-1} + $\frac{d\mu_k}{d\lambda}$) . Where μ_k is the k^{th} order central moment .

43) Let ,x is continuous random variable having spectrum ($-\infty <$ x$< \infty$) and distribution function F(x) . Show that the expected value of x is given by E(x) = $\int_0^{\infty}\{1 - F(x) - F(-x)\}dx$ provided x{1-F(x) – F (-x) } $\rightarrow$ 0 as x$\rightarrow \infty$.

44) The probability density function f(x) =$\frac{1}{2\theta}e^{-\left|\frac{x-\theta}{\theta}\right|}$, $-\infty < x < \infty$. Find the MGF of X . Hence , find mean and variable of X . Also Define MGF of a random variable X .

45) If the random variables x and y are connected by the linear relation ax + by +c=0 , then find the correlation co-efficient between x and y.

46) If the correlation co-efficient P(x,y) exists , then show that -1$\leq$ P(x,y) $\leq$ 1

47) If the independent random variables $x_1, x_2, \ldots, x_n$ all have the same distribution , then all of this is normally distributed.

48) If x and y be two random variables such that E(x^2), E(y^2) and E(x,y) exist , then prove that $\{E(xy)\}^2 \leq$ E (x) E(y)

49) The joint probablitity fun of two continuous random variables x and y where f(xy) = $\begin{cases} k(4-2x+y), 0<x<3, 2<y<4 \\ \quad\quad 0, \ elsewhere \end{cases}$ find p[x$< \frac{2}{y} < 3$] where k is a constant.

50) If x_1 is a B (n_1 ,p) and x_2 is a B (n_2 ,p) variates then show that x_1+ x_2 is also a bionomial variate .

51) If x and y are independent random variate, prove that they are un – correlated , but the converse is not true . Give an example to justify your answer

52) If $\rho = \rho$ (x, y) be the correlation coefficient between two random variables x and y then find the correlation coefficient between U = ax + by and V = CY where a, b, c are positive constants .

53) If $x_1^2 + x^2{}_2$ are independent x^2 variates having m and n degree of freedom respectively . Find the distribution of x_1^2 / x_2^2 .

54) If x has a t- distribution witn n – degrees of freedom then show that y = x^2 has an F (1 , n) distribution.

55) Prove that under certain conditions to be stated by you, the bionomial distribution tends to position distribution in the limit .

56) Find the characteristic function of a continuous random variable X with probability density function

$$f(x) = \{1-|X| \ , \quad |X| < 1$$
$$f(x) = \{0 \ , \quad elsewhere$$

57) If $X_1, X_2, X_3, \ldots, X_n$ are mutually independent standard normal variates , then find the mean value of min[$|X|_1, |X|_2, \ldots, |X|_n$].

58) If X is γ(n/2) variate, then show that Y=2X has a x^2 distribution with n degrees of freedom.

59) State and prove tehebycheff's Inequality for continuous and discrelevlt case .

60) If x_n is a bionomial (n,p)variate (0<p<1),show that the corresponding standardized variate $x_n^* = \frac{X_n - np}{\sqrt{npq}}$ is N(0,1) variate.

61) Show ,by tchebycheff's Inequality , that in 2000 throws with a coin the probability that the number of heads lies between 900 and 1100 is at least 19/20.

62) If X is a non negative random variate having mean m,prove that for any $\tau > 0$, P(X$\geq \tau m) \leq \frac{1}{\tau}$

63) If a random variable x possesses a finite3 second order moment and c is any fixed number , then show that for any $\in$<0. P($|X - c| \geq \in) \leq E[(X-c)^2]/\in^2$

64) Show that for the case of equal components the central limit theorem implies the law of large numbers.

65) Prove that the condition probability satisfies the axioms of probability.

66) A secretary writes four letters and the corresponding address on envelopes. He inserts the letters in the envelopes at random irrespective of the address .What is the probability that all letters are wrongly placed ? Also find the probability that one letter is placed in right envelope**.**

67) Let X and Y are independent variates , each uniformly distributed over the interval (0,1). Find the probability that the greater of X and Y is less then a fixed number K(0<K<1) .

68) Explain the random sample and write the difference between sampling distribution and distribution of sample.

69) Find the sampling distribution of the $atics = \frac{\sqrt{n}(\bar{x} - m)}{s}$, where $\overline{x}$ is the sample mean and(n-1)$s^2 = \sum_{i=1}^{n}(\text{xi}-\overline{x})^2$.($x_1, x_2, \ldots, x_n$represent sample of size n from a normal population of mean m and variance v^2.)

70) Show that regression coefficients independent of change of origin but depends on change of scales of variable.

71) Apply the method of likelihood ratio testing to $develope\ a$ method of testing the $hypothens$ H_0:$p=p_0$for binomial (n, p) population, when n is known and large.

72) Define likelihood function. Find the maximum likelihood estimate of the parameter λ of a continuous population having the density function f(x) =$\lambda\ x^{\lambda-1}$, 0<x<1whereλ>0.

73) In a group of 10 students , a dull students secured 25 marks below the average marks of the other students. Prove that the standard deviation of marksof all students is

at least7.5.If this standard deviation is actually 12, find the standard deviation when the dull student is left out.

74) The heights in inches of 7 students of a collage,chosen at random were as follows:

65.3, 68.4, 68.2, 64.2, 64.5, 61.5

Computer 95% confidence intervals for the mean and standard deviation of the population of height of students of the collage, assuming it to be normal.

Given p (t>2.447) = 0.025

P(x^2>1. 218) = 0.975

P(x^2>14.626)=0.025

For 6 degree of freedom.

75) Distinguish between a population and a sample.

76) If r be the sample correlation co-efficient of a $bivariate$ sample (x_1, y_1),(x_2,y_2),…,(xn, yn), then prove that -1$\leq r \leq 1$,also prove that the sample correlation co-efficient is independent of the unit of measurement and of the choice of origin. Given signification for r=0

77) If { A_n } be a monotonic sequence of events, then show that

P ($\lim A_n$) = $\lim P(A_n)$

78) The joint probability density function of the random variables X and Y is given by f(x,y) $=\begin{cases} x+y, 0<x<1 \\ 0, \ elsewhere \end{cases}$

Find the distribution of X+Y

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